REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	January 19, 2001	Final, May 19, 1997 - January 18, 2001

4. TITLE AND SUBTITLE
Nonlinear Coupling Between Control and Dynamic Parameters in Flexible Multibody Dynamics

5. FUNDING NUMBERS
Award # DAAG55-97-1-0303

6. AUTHOR(S)
Ahmed A. Shabana

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Illinois at Chicago
Chicago, Illinois

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
ARO 35711.29-EG

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT	12 b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words)

This report summarizes the main results obtained in the ARO funded research project performed at the University of Illinois at Chicago. The objectives of this research project were to provide a comprehensive study and to develop new computational methodologies in the area of mechanics, and control of constrained deformable bodies as applied to large scale flexible mechanical systems. In this research project, a new finite element procedure, the *absolute nodal coordinate formulation*, was developed. This new procedure can be used for the large deformation and rotation analysis of flexible multibody systems. It leads to exact modeling of the rigid body dynamics, and to a constant mass matrix for the finite elements in two- and three-dimensional applications. As a consequence, the vector of Coriolis and centrifugal forces is identically equal to zero. The new formulation captures the effect of the geometric centrifugal stiffness and accounts for the effect of the elastic nonlinearities. Several large deformation multibody problems were examined, and the results obtained using the new procedure were compared with the results obtained using existing finite element formulations. The results obtained in this research project are documented in several publications listed in this report.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Multibody dynamics, Controls, Finite elements, Large deformations, Finite rotations.	10
	16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

Enclosure 1

2001022 037

NONLINEAR COUBLING BETWEEN CONTROL AND DYNAMIC PARAMETERS IN FLEXIBLE MULTIBODY DYNAMICS

FINAL PROGRESS REPORT

Author

Ahmed A. Shabana

January 19, 2001

U.S. ARMY RESEARCH OFFICE

CONTRACT/GRANT NUMBER: **DAAG55-97-1-0303**

INSTITUTION: University of Illinois at Chicago

TABLE OF CONTENTS

1. PROBLEM STATEMENT

The objective of this research project is to provide a comprehensive study and to develop new computational methodologies in the area of mechanics and control of constrained deformable bodies as applied to large scale flexible mechanical systems. The main objectives of the project can be summarized as follows:

(1) Develop a classification of the methodologies used in the dynamics and control of flexible multibody systems in order to define the basic assumptions underlying each method. This classification will be used to establish guidelines for selecting the appropriate formulation for a given application. The new classification will be based on the ability and accuracy of the method in representing the rigid body dynamics, the degree of accuracy of each method in representing the coupling between the rigid body and deformation degrees of freedom, and the basic assumptions underlying each method.

(2) Use the new classification to define for the first time the relationship between different methods used in flexible multibody dynamics, and examine the effect of the linearization resulting from the use of the incremental finite element formulations on the definition of forces in flexible multibody dynamics.

(3) Develop new and efficient computational finite element procedures for the nonlinear dynamic analysis of constrained flexible multibody systems. These new computational methods will lead to accurate representation of the large displacements and can be used to circumvent the limitations of existing methods used in the nonlinear dynamic simulation of constrained flexible multibody systems.

(4) Examine the fundamental relationship between the deformable body coordinate system, the boundary conditions, the resonance conditions, and control forces in the dynamics of constrained deformable bodies.

(5) Demonstrate the use of the new procedures and formulations developed in this research project using flexible multibody applications.

The objectives of this research project have not been modified from the original application.

2. MAIN RESULTS

Since the start of this project in May of 1997, the following tasks have been completed:

1. A review of past and recent developments of the dynamics of flexible multibody

systems is presented [1]. The objective was to review the basic approaches used in the computer aided kinematic and dynamic analysis of flexible mechanical systems, and to identify future directions in this research area. Among the formulations reviewed are the floating frame of reference formulation, the finite element incremental methods, large rotation vector formulations, the finite segment methods, and the linear theory of elasto-dynamics. Linearization of the flexible multibody equations that results from the use of the incremental finite element formulations is demonstrated and discussed [1,2].

2. An absolute nodal coordinate formulation is developed for the dynamic analysis of flexible bodies that undergo large rotations and deformations [1-8]. In this absolute nodal coordinate formulation, no infinitesimal or finite rotations are used as nodal coordinates, instead global displacements and slopes are used as the nodal coordinates. Using this set of absolute coordinates, exact representation of the rigid body dynamics can be obtained, and an arbitrary rigid body motion leads to a zero strain in the finite element. The absolute nodal coordinate formulation leads to a simple constant mass matrix. Furthermore, using this formulation, beam and plate elements can be considered as isoparametric elements.

3. The absolute nodal coordinate formulation was applied to several flexible multibody system applications. The results obtained were compared with the results obtained using the floating frame of reference formulation, which is the most widely used method in flexible multibody simulations. Excellent agreement between the two methods was observed in the case of small deformation problems [9].

4. The effect of the choice of the element coordinate system in the non-incremental finite element absolute nodal coordinate formulation [12-13] was investigated. This formulation leads to a constant and symmetric mass matrix and highly non-linear elastic forces. It was demonstrated in this project that different element coordinate systems which are used for the convenience of describing the element deformation lead to similar results as the element size is reduced [12]. In order to demonstrate this fact, two element frames are used in this study; the *pinned* and the *tangent frames*. The pinned frame has one of its axes passes through two nodes of the element, while the tangent frame is rigidly attached to one of the end points of the element. Numerical results obtained in this project using these two frames are found to be in a good agreement as the element size decreases.

5. The equivalence of the finite element formulations used in flexible multibody dynamics has been the subject of extensive study in this research project. This equivalence was used to address several fundamental issues related to the deformations, flexible body coordinate systems, and the geometric centrifugal stiffening effect. The equivalence of the inertia forces used in different finite element formulations was previously established [6]. The relationship between the

coordinates used in the absolute nodal coordinate formulation and the coordinates used in the floating frame of reference formulation was developed. This relationship is used to obtain the highly nonlinear expression of the strain energy used in the absolute nodal coordinate formulation from the simple energy expression used in the floating frame of reference formulation. It was shown in this study [12] that the source of the nonlinearity in the elastic forces is due to the finite rotation of the element. The results obtained clearly demonstrate that the instability observed in high speed rotor analytical models due to the neglect of the geometric centrifugal stiffening is not a problem inherent to a particular finite element formulation. Such a problem can be only avoided by considering the known linear effect of the geometric centrifugal stiffening or by using a nonlinear elastic model as recently demonstrated. Fourier analysis of the solutions obtained was used to shed new light on the fundamental problem of the choice of the deformable body coordinate system in the floating frame of reference formulation [12, 13].

6. Two procedures for the computer implementation of the non-incremental absolute nodal coordinate formulation were proposed for flexible multibody applications [10]. These two procedures utilize the fact that the mass matrix obtained using the absolute nodal coordinate formulation is constant in order to obtain an optimum sparse matrix structure. In the first procedure, a QR decomposition of a modified constant connectivity Jacobian matrix is obtained for the deformable body. A constant velocity transformation is used to obtain an identity generalized inertia matrix associated with the second derivatives of the generalized coordinates, thereby minimizing the number of non-zero entries of the coefficient matrix that appears in the augmented Lagrangian formulation of the equations of motion of the flexible multi-body systems. In the second procedure, a Cholesky decomposition of the constant mass matrix is used to define another constant velocity transformation matrix. This alternate procedure, which has the same computational advantages as the one based on the QR decomposition, leads to a square velocity transformation matrix.

7. The use of the non-incremental finite element absolute nodal coordinate formulation in the analysis of large deformation and rotation problems was demonstrated using several structural and multi-body applications [8, 15]. The results obtained are compared with the results obtained using existing methods. These results demonstrate that the absolute nodal coordinate formulation can be considered as the basis for developing a new generation of general purpose flexible multi-body computer codes that can be used in the analysis of small and large deformation problems.

8. The performance of the new absolute nodal coordinate formulation developed in this investigation was compared to the incremental finite element methods using several structural and mechanical system applications. Numerical results showed that there is a good agreement between the results obtained using the new method and the

results obtained using the widely used incremental finite element methods when structural systems are analyzed. On the other hand, several large rotation and deformation multibody problems showed that the absolute nodal coordinate formulation is more efficient and leads to a better accuracy [14,15,19].

9. New and simple elastic force models were developed using the absolute nodal coordinate formulation. Four new models that employ a nonlinear strain-displacement relationship were presented. These new force models were used to develop a general purpose flexible multibody computer algorithm. A comparison with the model previously developed shows a significant reduction in computer time and improvement in the accuracy using a smaller number of elements [13,16,18].

10. The problem of the geometric centrifugal stiffening in rotating beams was examined in details. It was shown that the absolute nodal coordinate formulation automatically accounts for the geometric centrifugal stiffening. This is an important feature of the new formulation, particularly when rotor systems are analyzed. It was demonstrated that the absolute nodal coordinate formulation can efficiently be used to predict the natural frequencies and mode shapes of rotating beams. Numerical results are compared with the results previously published [20].

11. It was demonstrated that the absolute nodal coordinate formulation can lead to a constant mass matrix for three-dimensional elements by increasing the number of nodal coordinates. Using this important property of the new formulation, an efficient procedure for the solution of the large deformation in spatial flexible multibody systems can be developed using Cholesky coordinates, as previously demonstrated in the planar case [17].

3. LIST OF PUBLICATIONS

The formulations and procedures developed, and the results obtained in this research project are documented in the following publications:

(a) Refereed Journal Publications

1. Shabana, A.A., and Christensen, A.P., "Three Dimensional Absolute Nodal Coordinate Formulation: Plate Problem", *International Journal for Numerical Methods in Engineering*, Vol. 40, No. 15, 1997, pp. 2775-2790.

2. Shabana, A.A., "Flexible Multibody Dynamics: Review of Past and Recent Development", *Journal of Multibody System Dynamics*, Vol. 1, No. 2, 1997, pp.189-222.

3. Shabana, A.A., "Definition of the Slopes and Finite Element Absolute Nodal Coordinate Formulation", *Journal of Multibody System Dynamics*, Vol. 1, No. 3, 1997, pp. 339-348.

4. Shabana, A.A., and Schwertassek, R.A., "Equivalence of the Floating Frame of Reference Approach and Finite Element Formulations", *International Journal of Nonlinear Mechanics*, Vol. 33, No. 3 , 1998, pp. 417-432.

5. Choi, J.H., Lee, H.C., and Shabana, A.A.,"Spatial Dynamics of Multibody Tracked Vehicles, Part I: Spatial Equations of Motion", *Journal of Vehicle System Dynamics*, Vol. 29, No. 1, January 1998, pp. 27-49.

6. Lee, H.C., Choi, J.H., and Shabana, A.A., "Spatial Dynamics of Multibody Tracked vehicles, Part II: Contact Forces and Simulation Results", *Journal of Vehicle System Dynamics*, Vol. 29, No. 2, February 1998, pp. 113-137.

7. Choi, J.H., Campanelli, M., Shabana, A.A., and Wehage, R.A., "Approximation Methods in the Nonlinear Analysis of Multibody Tracked Vehicles", *Journal of Vehicle System Dynamics*, Vol. 29, No. 3, March 1998, pp. 181-211.

8. Christensen, A.P., and Shabana, A.A., "Exact Modeling of the Spatial Rigid Body Inertia Using the Finite Element Method", *ASME Journal of Vibration and Acoustics*, Vol. 120, No. 3, July 1998, pp. 650-657.

9. Shabana, A.A., Hussien, H.A., and Escalona, J.L., "Application of the Absolute Nodal Coordinate Formulation to Large Rotation and Large Deformation Problems", *ASME Journal of Mechanical Design*, Vol. 120, No. 2, June 1998, pp. 188-195.

10. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "Application of the Absolute Nodal Coordinate Formulation to Multibody System Dynamics", *Journal of Sound and Vibration*, Vol. 214, No. 5, July 1998, pp. 833-851.

11. Shabana, A.A., "Computer Implementation of the Absolute Nodal Coordinate Formulation for Flexible Multibody Dynamics", *Nonlinear Dynamics*, Vol. 16, No. 3, July 1998, pp. 293-306.

12. Campanelli, M., Shabana, A.A., and Choi, J.H., "Chain Vibration and Dynamic Stress in Three-Dimensional Multibody Tracked Vehicles", *Multibody System Dynamics*, Vol.2, No. 3, September 1998, pp. 277-316.

13. Yakoub, R.Y., and Shabana, A.A., "Use of Cholesky Coordinates and the Absolute Nodal Coordinate Formulation in the Computer Simulation of Flexible Multibody Systems", *Nonlinear Dynamics*, Vol.20, No. 3, 1999, pp.267-282.

14. Schwertassek, R., Wallrap, O., and Shabana, A.A., "Flexible Multibody Simulation and Choice of Shape Functions", *Nonlinear Dynamics*, accepted for publication.

15. Berzeri, M., Campanelli, M., and Shabana, A.A., "Definition of the Elastic Forces in the Finite Element Absolute Nodal Coordinate Formulation and the Floating Frame of Reference Formulation", *Multibody System Dynamics*, accepted for publication.

16. Berzeri, M., and Shabana, A.A., "Development of Simple Models for the Elastic Forces in the Absolute Nodal Coordinate Formulation", *Sound and Vibration*, Vol. 235, No. 4, 2000, pp. 539-565.

17. Campanelli, M., Berzeri, M., and Shabana, A.A., "Performance of the Incremental and Non-Incremental Finite Element Formulations in Flexible Multibody Problems", *ASME Journal of Mechanical Design*, accepted for publication.

18. Omar, M.A., and Shabana, A.A., "A Two-Dimensional Shear Deformable Beam Element for Large Rotation and Deformation Prblems", *Sound and Vibration*, accepted for publication.

19. Shabana, A.A., Berzeri, M., and Sany, J.R., "Numerical Procedure for the Simulation of Wheel/Rail Contact Dynamics", *ASME Journal of Dynamic Systems, Measurement, and Control*, accepted for publication.

(b) Refereed Conference Proceedings

20. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "An Absolute Nodal Coordinate Formulation for the Dynamic Analysis of Flexible Multibody Systems", Proceedings of the 12th European Simulation Multi-conference, June 16-19, 1998, Manchester, United Kingdom.

21. Campanelli, M., Shabana, A.A., and Choi, J.H., "Three-Dimensional Stress Analysis of the Chains of Tracked Vehicles", Proceedings of the ASME 1998 Dynamic Systems and Control Division, ASME 1998 International Mechanical Engineering Congress & Exposition, Anaheim, California, November 15-20, 1998, DSC-Vol. 64, pp. 645-655.

22. Yakoub, R.Y., and Shabana, A.A., "A Numerical Approach for Solving Flexible Multibody Systems Using the Absolute Nodal Coordinate Formulation", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las Vegas, Nevada, September 12-15, 1999.

23. Berzeri, M., Campanelli, M., and Shabana, A.A., "Elastic Forces and Coordinate Systems in the Finite Element Formulations", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las

Vegas, Nevada, September 12-15, 1999.

24. Berzeri, M., Campanelli, M., and Shabana, A.A.,"Incremental Finite Element Formulations and Flexible Multibody Dynamics", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las Vegas, Nevada, September 12-15, 1999.

25. Shabana, A.A., "Performance of Non-Linear Finite Element Formulations in Flexible Multibody Computer Simulations", Proceedings of the NATO Advanced Research Workshop on *Computational Aspects of Nonlinear Structural Systems with Large Rigid Body Motion*, Pultusk, Poland, July 2-7, 2000, pp.219-230.

(c) Technical Reports

26. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "Application of the Absolute Nodal Coordinate Formulation to Multibody System Dynamics", Technical Report # MBS97-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, 1997.

27. Shabana, A.A., "A QR Decomposition Method for Flexible Multibody Dynamics", Technical Report # MBS97-2-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, 1997.

28. Berzeri, M., Campanelli, M., and Shabana, A.A., "Definition of the Elastic Forces in the Finite Element Formulations", Technical Report # MBS98-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, November, 1998.

29. Shabana, A.A., Berzeri, M., and Campanelli, M., "Examples of Large Deformation Results Obtained Using the Absolute Nodal Coordinate Formulation", Technical Report # MBS99-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

30. Campanelli, M., Berzeri, M., and Shabana, A.A., "Comparison Between the Absolute Nodal Coordinate Formulation and Incremental Procedures",Technical Report # MBS99-2-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

31. Berzeri, M., and Shabana, A.A., "A Continuum Mechanics Approach for Formulating the Elastic Forces in the Absolute Nodal Coordinate Formulation",Technical Report # MBS99-3-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

32. Berzeri, M., and Shabana, A., "A Finite Element Study of the Geometric Centrifugal Stiffening Effect ", Technical Report # MBS99-5-UIC, Department of Mechanical Engineering, The University of Illinois at Chicago, December 1999.

33. Shabana, A.A., and Yakoub, R.Y., "An Isoparametric Three Dimensional Beam Element Using the Absolute Nodal Coordinate Formulation",Technical Report # MBS00-1-UIC, Department of Mechanical Engineering, The University of Illinois at Chicago, February 2000.

34. Omar, M.A., and Shabana, A.A. "Development of a Shear Deformable Element Using the Absolute Nodal Coordinate Formulation",Technical Report # MBS00-3-UIC, Department of Mechanical Engineering, The University of Illinois at Chicago, May 2000.

35. Mikkola, A.M., and Shabana, A.A. "A Large Deformation Plate Element for Multibody Applications",Technical Report # MBS00-6-UIC, Department of Mechanical Engineering, The University of Illinois at Chicago, October 2000

(d) Presentations

36. Shabana, A.A., "Large Amplitude Nonlinear Vibration of Multibody Systems", Presented at the 1999 ASME Applied Mechanics & Materials Conference, Blacksburg, Virginia, June 27-30, 1999.

37. Shabana, A.A., "Use of the Absolute Nodal Coordinate Formulation in Modeling Flexible Multibody Systems", Presented at the 20th International Congress of Theoretical and Applied Mechanics (ICTAM 2000), Chicago, Illinois, August 27-September 2, 2000.

4. SCIENTIFIC PERSONNEL

The list of all researchers who participated and supported by this research project is as follows:

1. Ahmed A. Shabana, Principal Investigator
2. Marcello Campanelli, Graduate student
3. Hussien Hussien, Graduate student
4. Marcello Berzeri, Graduate student
5. Refaat Yakoub, Graduate student
6. Chen Chen Yao, Graduate student
7. Pinsopin Unnat, Graduate student

The degrees awarded are as follows:

1. Marcello Campanelli, Ph.D., 1998 (currently employed by Lord Corp.)
2. Hussien Hussien, Ph.D., 1998 (currently employed by Ford Motor Co.)
3. Marcello Berzeri, Ph.D., 2000 (currently employed by Ford Motor Co.)
4. Refaat Yakoub, Ph.D., expected 2001

5. REFERENCES

1. Shabana, A., "Flexible Multibody Dynamics: Review of Past and Recent Development", *Journal of Multibody System Dynamics*, Vol. 1, No. 2, 1997, pp.189-222.

2. Shabana, A., and Christensen, A., "Three Dimensional Absolute Nodal Coordinate Formulation: Plate Problem", *International Journal for Numerical Methods in Engineering*, Vol. 40, No. 15, 1997, pp. 2775-2790.

3. Shabana, A., "Definition of the Slopes and Finite Element Absolute Nodal Coordinate Formulation", *Journal of Multibody System Dynamics*, Vol. 1, No. 3, 1997, pp. 339-348.

4. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "Application of the Absolute Nodal Coordinate Formulation to Multibody System Dynamics", Technical Report # MBS97-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, 1997.

5. Shabana, A., "A QR Decomposition Method for Flexible Multibody Dynamics", Technical Report # MBS97-2-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, 1997.

6. Shabana, A.A., and Schwertassek, R.A., "Equivalence of the Floating Frame of Reference Approach and Finite Element Formulations", *International Journal of Nonlinear Mechanics*, Vol. 33, No. 3 , 1998, pp. 417-432.

7. Christensen, A.P., and Shabana, A.A., "Exact Modeling of the Spatial Rigid Body Inertia Using the Finite Element Method", *ASME Journal of Vibration and Acoustics*, Vol. 120, No. 3, July 1998, pp. 650-657.

8. Shabana, A.A., Hussien, H.A., and Escalona, J.L., "Application of the Absolute Nodal Coordinate Formulation to Large Rotation and Large Deformation Problems", *ASME Journal of Mechanical Design*, Vol. 120, No. 2, June 1998, pp. 188-195.

9. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "Application of the Absolute Nodal Coordinate Formulation to Multibody System Dynamics", *Journal of Sound and Vibration*, Vol. 214, No. 5, July 1998, pp. 833-851.

10. Shabana, A.A., "Computer Implementation of the Absolute Nodal Coordinate Formulation for Flexible Multibody Dynamics", *Nonlinear Dynamics*, Vol. 16, No. 3, July 1998, pp. 293-306.

11. Escalona, J.L., Hussien, H.A., and Shabana, A.A., "An Absolute Nodal Coordinate

Formulation for the Dynamic Analysis of Flexible Multibody Systems", Proceedings of the 12th European Simulation Multi-conference, June 16-19, 1998, Manchester, United Kingdom.

12. Berzeri, M., Campanelli, M., and Shabana, A.A., "Definition of the Elastic Forces in the Finite Element Formulations", Technical Report # MBS98-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, November, 1998.

13. Berzeri, M., Campanelli, M., and Shabana, A.A., "Definition of the Elastic Forces in the Finite Element Absolute Nodal Coordinate Formulation and the Floating Frame of Reference Formulation", *Multibody System Dynamics*, accepted for publication.

14. Shabana, A.A., Berzeri, M., and Campanelli, M., "Examples of Large Deformation Results Obtained Using the Absolute Nodal Coordinate Formulation", Technical Report # MBS99-1-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

15. Campanelli, M., Berzeri, M., and Shabana, A.A., "Comparison Between the Absolute Nodal Coordinate Formulation and Incremental Procedures",Technical Report # MBS99-2-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

16. Berzeri, M., and Shabana, A.A., "A Continuum Mechanics Approach for Formulating the Elastic Forces in the Absolute Nodal Coordinate Formulation",Technical Report # MBS99-3-UIC, Department of Mechanical Engineering, University of Illinois at Chicago, March, 1999.

17. Yakoub, R.Y., and Shabana, A.A., "A Numerical Approach for Solving Flexible Multibody Systems Using the Absolute Nodal Coordinate Formulation", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las Vegas, Nevada, September 12-15, 1999.

18. Berzeri, M., Campanelli, M., and Shabana, A.A., "Elastic Forces and Coordinate Systems in the Finite Element Formulations", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las Vegas, Nevada, September 12-15, 1999.

19. Berzeri, M., Campanelli, M., and Shabana, A.A.,"Incremental Finite Element Formulations and Flexible Multibody Dynamics", Proceedings of the 1999 ASME Seventeenth Biennial Conference on Mechanical Vibration and Noise, ASME Design Technical Conferences, Las Vegas, Nevada, September 12-15, 1999.

20. Berzeri, M., and Shabana, A., "A Finite Element Study of the Geometric Centrifugal Stiffening Effect ", Technical Report # MBS99-5-UIC, Department of Mechanical Engineering, The University of Illinois at Chicago, December 1999.